AP Calculus BC

Unit 3 - FRQ

1)

a) $g(x) = f(2x - x^2)$

$g'(x) = f'(2x - x^2)(2 - 2x)$

$g'(-1) = f'(-3)(4)$

$= (-2)(4) = -8$

b) $g''(x) = f'(2x - x^2)(-2) + f''(2x - x^2)(2 - 2x)(2 - 2x)$

$g''(0) = f'(0)(-2) + f''(0)(4) = 0$

$-8 + f''(0)(4) = 0$

$f''(0) = 2$

c) Since g is differentiable it is also continuous, IVT applies:

$g(0) = f(0) = -1 < 2$

$g(3) = f(-3) = 5 > 2$

$g(c) = 2$ in $(0,3)$

d) $h'(x) = \dfrac{f(4) - f(0)}{4 - 0}$

$h'(x) = \dfrac{8}{4} = 2$

$h'(x) = 2$ @ $x = 0.351$ or $x = 0.352$

2)

a) $2xy + y^2 = 8$

$2xy' + 2y + 2yy' = 0$

$2xy' + 2yy' = -2y$

$y' = \dfrac{-y}{x+y}$

b) $\left.\dfrac{dy}{dx}\right|_{(1,2)} = \dfrac{-2}{3}$

$y - 2 = -\frac{2}{3}(x-1)$

c) $\dfrac{d^2y}{dx^2} = \dfrac{(x+y)(-y') - (-y)(1+y')}{(x+y)^2}$

$\left.\dfrac{d^2y}{dx^2}\right|_{(1,2)} = \dfrac{(3)(-\frac{2}{3}) - (-2)(\frac{1}{3})}{(9)}$

d)

$y(\frac{7}{2}) = 1$	$(y^{-1})(1) = \frac{7}{2}$
$\left.\dfrac{dy}{dx}\right|_{(\frac{7}{2},1)} = \dfrac{-1}{9/2} = -\dfrac{2}{9}$	$(y^{-1})'(1) = -\frac{9}{2}$

3) a) $g(x) = \dfrac{\ln x}{f(x^3)}$

$$g'(x) = \frac{f(x^3)\cdot\frac{1}{x} - \ln x \cdot f'(x^3)\cdot 3x^2}{\left[f(x^3)\right]^2}$$

$$g'(2) = \frac{f(8)\cdot\frac{1}{2} - \ln 2 \cdot f'(8)\cdot 12}{\left[f(8)\right]^2} = \boxed{\frac{3-4\ln 2}{36}}$$

b) $h(x) = f(f(-3x))$

$$h'(x) = f'(f(-3x))\cdot f'(-3x)\,(-3)$$

$$h'(-1) = f'(f(3))\cdot f'(3)\,(-3)$$

$$= f'(2)\cdot\left(-\tfrac{2}{5}\right)(-3)$$

$$= \left(\tfrac{3}{2}\right)\left(-\tfrac{2}{5}\right)(-3)$$

c) $k(x) = f(x)\cdot \arctan\left(\frac{x}{3}\right)$

$$k'(x) = f(x)\cdot\frac{\frac{1}{3}}{1+\left(\frac{x}{3}\right)^2} + f'(x)\arctan\left(\tfrac{x}{3}\right)$$

$$k'(-3) = f(-3)\cdot\frac{\frac{1}{3}}{1+1} + f'(-3)\arctan(-1)$$

$$= (-9)\left(\tfrac{1}{6}\right) + \left(-\tfrac{7}{2}\right)\left(-\tfrac{\pi}{4}\right)$$